

When Children Rule: Parenting in Modern Families

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Abstract

During the 20th century there was a secular transformation within American families from a household dominated by the father to a more egalitarian one in which the wife and the children have been empowered. This transformation coincided with two major economic and demographic changes, namely the increase in economic opportunities for women and a decline in family size. To explain the connection between these trends and the transformation in family relationships we develop a novel model of parenting styles that highlights the importance of competition within the family. The key intuition is that the rise in relative earnings of wives increased competition between spouses for the love and affection of their children while the decline in family size reduced competition between children for resources from their parents. The combined effect has empowered children within the household and allowed them to capture an increasing share of the household surplus over the past hundred years.

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1 Introduction

Over the course of the 20th century there has been a secular shift in the nature of relationships within American families. In the early 1900's the typical father was the dominant head of the household and commanded the respect of his wife and children. The family has since become a more egalitarian organization. We argue that this shift in the distribution of power within the household is intimately connected to economic and demographic changes that occurred during the same period. More specifically, the decline in family size and the improvement in the economic opportunities for women have empowered children within the household and transformed parenting styles. We document trends in the earnings gap between husbands and wives, family size and parenting styles and explain the connection between the three using a novel model of parenting that highlights the role of competition within the family. In particular, we conceptualize the family as an environment in which individuals compete with each other over scarce resources (material and emotional). A decline in family size reduces competition between siblings for resources from their parents while an increase in female earnings leads to greater competition between parents for the love and affection of their children. The combined effect improves the bargaining position of children and explains why we moved towards a family in which the children rule.

We develop a noncooperative model of the family in which parenting styles emerge as a Nash equilibrium. The utility of the children is determined by their own behavior and the material investment they receive from their parents, for which they compete with their siblings. We model this competitive process as an auction in which children offer good behavior (behavior preferred by the parents but not by the children) in return for material investment from the parents. The children are willing to be well behaved when the investment they can potentially receive is large. Parents care about the behavior of their children and their own material consumption. Both parents have the ability to influence the children through material investment using their own auction and we allow child behavior to be part public and part private good. This captures the idea that some types of children behavior such as effort in school are public goods—i.e., a good enjoyed by both parents—while other behaviors such as love and affection toward a parent are private goods. Likewise, we also allow for material consumption to be partially a private and public good. Each parent decides how to allocate their own earnings between material consumption and investment in the children. In equilibrium, the allocation of resources within the household is fully

determined, i.e., parental investments are allocated among siblings, the behavior of the children is defined toward each parent, material consumption and investments are determined for each parent.

We use the model to explain the relationship between three empirical trends: a decline in the earnings gap between husbands and wives, a decline in the number of children per family and an increase in the share of the household surplus captured by the children. We show that the payoff of each child is decreasing in the number of siblings and increasing in the investments made by the parents. Moreover, we show that parental investment is independent of family size and deduce conditions under which parental investments increase as the gap between male and female earnings close. Taken together, this implies that a decline in the earnings gap and family size should increase the share of the household surplus captured by the children. Two mechanisms are behind these results. First, as there are fewer children in the household, competition for parental investments is less intense, which decreases the bids offered by each child. Second, when the behavior of the children is relatively more of a private good (relative to the degree to which material consumption is a private good), the spouse with lower earnings prefers to free-ride off the material consumption of the higher earning spouse and instead focuses all of his or her resources on influencing the behavior of the children. A reduction in the income gap between the spouses increases total parental investment because the lower earning spouse is now able to pour more resources into the fight to control the behavior of the children.

In the baseline model the only way in which parents can influence the children is through parental investment. In reality, parents also use punishment in order to impose their will and discipline the children. In an extension of the baseline model we formally introduce punishment and use the model to explain the decline in corporal punishment in the 20th century. In our model punishment shapes the behavior of the child but causes disutility for both the parent and the child. In addition to preserving the key findings above, the equilibrium produces several interesting comparative statics related to the use of punishment. First, as total household income increases, punishment declines because parents now have more material resources to influence the behavior of their children and do not have to resort to coercion. Second, a reduction in the gap between spousal earnings leads to a more intense use of punishment. Third, a decrease in the number of children induces a less intense use of punishment if the elasticity of the expected behavior of the children in the auctions with respect to the number of children is relatively low. Thus, according to the model, empirical trends in total household income and family size should

reduce the incidence of corporal punishment while the decline in the earnings gap should have increased the use of corporal punishment. Given that corporal punishment did indeed decline during the period, we posit that the former two effects dominated the latter.

This paper relates to the extensive literature on parental investments in children. The seminal work of Becker and Tomes (1979 and 1986) explore the role of the family in economic mobility. One strand of subsequent literature studies the nature of skill formation and has looked at: existence of critical investment periods, role of parental skills, multidimensional nature of skills, imperfect information of the skill formation process and dynamic complementarities in skill production (see Heckman and Cunha 2008, Cunha et al. 2010, Cunha et al. 2013 and Todd and Wolpin 2003, 2007). Other papers have documented the importance of parental time inputs (Del Boca 2014) and the role of the marriage market (Gayle et al. 2014). The consensus in the literature is that characteristics formed during childhood play a major role in predicting future earnings (Keane and Wolpin 1997 and Huggett et al. 2011), individuals are more malleable at younger ages (Cunha et al. 2006), and parental inputs have significant long-term impacts through skill formation. See Heckman and Mosso (2014) and Francesconi and Heckman (2016) for comprehensive reviews of this literature. Our paper seeks to contribute to this literature by explaining changes in parental investments and parenting styles with an economic model in which parenting styles are shaped by competitive forces within the household.

Much of the previously mentioned literature on parental investments focuses on the decisions of parents and treats children as passive agents. Closer to this paper are the works that treat parenting as the outcome of a two-way strategic relationship between child and parents. Becker's (1974) rotten kid theorem is an early example of a model in which children are active agents and parents influence their behavior through transfers. Several more recent papers develop richer models of parent-child interactions in an attempt to understand more nuanced aspects of parenting styles. Lizzeri and Siniscalchi (2008) present a model in which children struggle with performing tasks. Parents decide whether or not to assist their children and shield them from failure or let them work on their own and risk failure but encourage learning. Burton et al. (2002), Cosconati (2013) and Akabayashi (2006) develop models in which children have greater discount rates and thus their optimal investment in accumulating human capital is lower than what their parents would choose. In response, the parents use carrots and sticks to incentivize their children. In a recent paper, Doepke and Zilibotti (2014) argue that parenting styles are influenced by

economic inequality and the returns to incumbency (working in the same profession as your parent). In their model parents are motivated by both altruism and paternalism and they influence the behavior of their children by shaping their preferences or restricting their choice sets. Our work is closely related to this recent literature in that we explicitly model both parents and children and allow parenting styles to arise out of a strategic interaction between children and their parents.

Our paper, however, differs from the existing literature by focusing on the role of competition within the household. Most models only consider families with one child. Of the models that do consider multiple children, siblings typically affect one another only through the resource constraint. There are a few notable exceptions that have explored parental responses to multiple children with different abilities. Behrman et al. (1982) studies how, depending on the preferences of the parent, parenting styles could amplify or mitigate innate differences across siblings. Hao et al. (2008) argue that parents tend to be stricter on older children in order to deter the younger siblings from misbehaving. Vogl (2013) illustrates how the institution of arranged marriage induces families to marry off their elder daughters at a younger age. While all of these papers explore interesting interactions between siblings, they do not explicitly model competition between family members. Rather, these models consider how parents would react optimally given multiple and heterogeneous children in the family. A key contribution of this paper is to highlight the importance of competition within the household.

Bernheim et al. (1985) present a model in which strategic interactions between children play a prominent role. Specifically, they develop a model of intergenerational transfers in which parents influence the behavior of their children by threatening to disinherit them. Their model differs in two important ways from ours. First, they describe interactions between children as strategic but not competitive. The key intuition in their model is that threats of disinheritance are only credible when there is more than one child because with one child parents have no alternative allocation of inheritance. In contrast, our model explicitly addresses the idea that children compete with each other for resources from the parents: better own behavior leads to greater expected investment while better behavior of other children leads to lower expected investment. Second, their goal is to explain patterns in inheritance. Our model aims to capture parenting styles and envisions transfers as more instantaneous measures of parental investment. Focusing on this types of transfer may be a more realistic interpretation since Berhman and Rosenzweig (2004) find “*that there are not significant sibling differences in bequests, despite the fact that the siblings*

differ significantly in their earning abilities and the amount of time they spend with parents prior to their parent's death."

The rest of the paper is organized as follows. Section 2 documents the main trends in economic opportunities for women, family size and parenting styles in the U.S. over the last century. Section 3 develops a simple noncooperative model of intra-household decisions that stresses the role of competition among family members in the determination of the distribution of resources within the household; particularly, in the surplus accrued to children and parenting styles. Section 4 extends the model in order to introduce corporal punishment. Section 5 concludes.

2 Empirical Trends in Economic Opportunities for Women, Family Size and Parenting Styles

This section documents the trends in economic opportunities for women, family size and parenting styles in the U.S. over the last century. To provide a concrete example of these differences consider two famous families from entertainment. *Modern Family* is a contemporary television show that depicts the life of Phil and Claire Dunphy and their three kids Haley, Alex and Luke. While Claire is a stay-at-home mom, she is also a former marketing executive for a big hotel chain and is largely in charge of keeping both the kids and her husband in line. Phil on the other hand is “*a big kid at heart*” who refers to his parenting styles as “*pearenting*.” The relationships between these family members stand in stark contrast to those depicted in *One Man's Family*, a popular radio soap opera that aired between 1932 and 1959. The serial documented the life of Henry Barbour, his wife Fanny and their five children (Paul, Hazel, Clifford and Claudia, and Jack). Fanny, a housewife, maintains some level of control over domestic affairs but “*Henry is not, however, henpecked...[and] he remains the head of the Barbour clan and retains his dignity and the respect of his children and audience.*”¹

In the following subsections we document how the differences between these two families—at least with respect to the economic role of wives, the number of children and parenting styles—are representative national trends. We heavily rely on the work of other scholars but also build our own evidence from

¹The quotation for *Modern Family* is taken from the show's website and the quote describing Henry Barbour is taking from Sheppard (1964).

various sources. Since a key focus of our analysis is on how the relative earnings of the spouses and the number of children influence the distribution of resources between parents and children, we make a special effort to document that the trends apply to two parent households with children. Thus, when possible, we restrict the sample to households with married parents who both appear in the sample and have at least one child.²

2.1 Economic Opportunities for Women

Over the last century the earnings structure of the typical American household evolved from one in which the husband was the primary breadwinner to that of a dual earner household. Figures 1 and 2 use U.S. census data to trace out changes in labor force participation and the wife's contribution to household earnings.³ Figure 1 plots labor force participation for married men and women and figure 2 plots the earnings of the wife as a fraction of the combined earnings of the husband and wife. The measurement of labor force participation differs for years before and after 1940 and there is some debate as to how to compare the earlier and later data. The primary concern is that female labor force participation data from earlier years suffers from severe measurement error. However, in *Understanding the Gender Gap*, Claudio Goldin notes that “*despite various complications concerning the definition of paid and unpaid labor, and the distinction between the home and the work place, the labor force activity of all women advanced in a more or less continuous fashion from the early nineteenth century to present.*” Figures 1 and 2 clearly illustrate the transformational increases in the relative earning power of wives particularly during the latter half of the 20th century.

²There are other important changes to family structure that occurred during this time period –notably the rise of single mothers in the second half of the 20th century. However, an analysis of these changes is outside the scope of this paper.

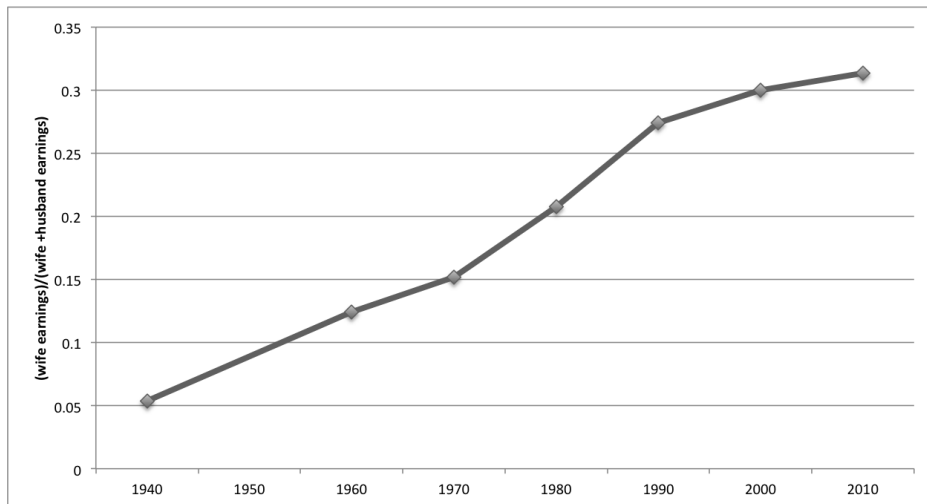
³The data are made available through Integrated Public Use Microdata Series (IPUMS) see Ruggles et al. (2015). In Figures 1-3 data for each year comes from 1% samples of the U.S. census with two exceptions. Data from 1980 is from the 5% sample because the 1% sample was not available while data from 2010 is from 5 years of the ACS because the 2010 census does not have labor force participation data. The sample in each year contains two-parent households with at least one child (individual under the age of 18) present in the household. Calculations in each year are completed using household weights provided through IPUMS.

Figure 1: Labor Force Participation Rates



Note: Data from 1940-2010 use the modern definition of labor for participation in which an individual works within a given reference week. Before 1940 an individual was recorded as being in the labor force if they reported any gainful occupation. As discussed above, the measures from the two time periods are not directly comparable.

Figure 2: Distribution of Earnings Between Spouses

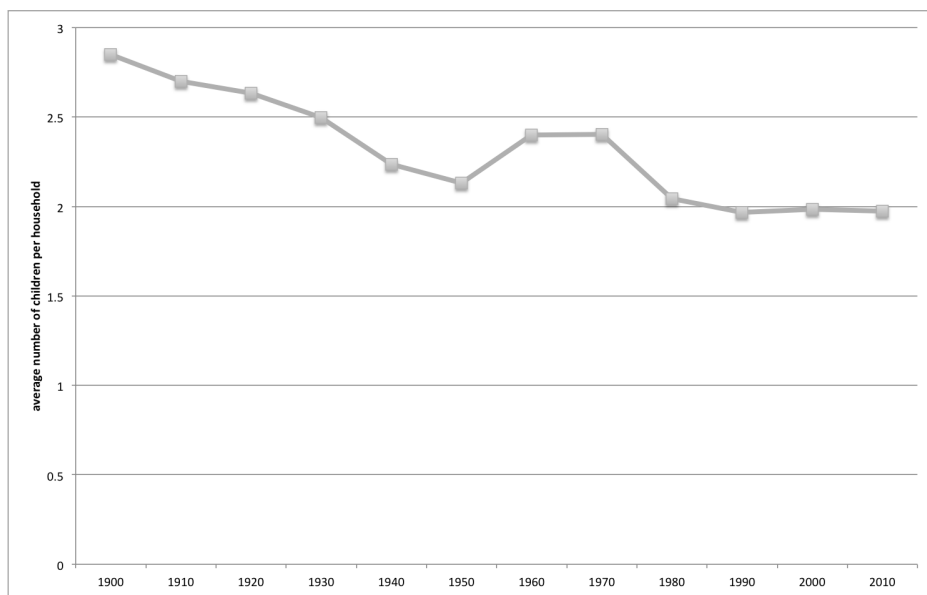


Note: Earnings are total pre-tax wage and salary income from the previous year. Households where either husband or wife are missing earnings data or both have zero earnings are dropped from the sample. 1950 is excluded due to data quality issues of the earnings variable.

2.2 Family Size

Compared to a century ago, parents today also have fewer children. Figure 3 shows the evolution in the average number of children per household with at least one child. The average number of children per household declined by about 0.9 between 1900 and 2010. This pattern is consistent with the existing evidence in the demographic literature, which has documented sustained declines in fertility starting in the 19th century (see, among others, Hacker 2003). In addition, Hobbs and Stoops (2002) use census data to document similar declines in family size in the 20th century.

Figure 3: Number of Children per Household



Note: Number of children is the number of own children residing in the household.

Data are from the same sources used to create figures 1 and 2.

2.3 Parenting Styles

Documenting trends in parenting styles is a difficult task. There is very little informative data collected in the early 20th century and the relationship between children and their parents is complex and multifaceted. Thus, measurement error is likely to be a serious issue. Despite these challenges, we argue that there is convincing evidence of a systematic change in parenting styles over the last hundred years. To support this claim we draw from various sources to show trends related to: attitudes towards children, discipline methods, expenditure patterns and time use. All of the patterns strongly suggest that a shift within the

household took place over the past century that has allowed children to capture a greater share of the household surplus.

A number of researchers have argued that general attitudes towards children have drastically changed during the 20th century. For example, Stearns (2003) argues that over the last two centuries parents have changed “*their ideas about children’s nature, attributing to it a greater sense of vulnerability and frailty.*” Stearns supports these claims by citing parenting advice, developments in medicine (such as the focus on children’s posture), views of organizations and articles in newspapers and magazines. In addition, he documents how these changes in attitudes have coincided with changes in behaviors providing examples such as: parental discipline methods, grade inflation, decline in child labor and a growing obsession of entertaining children. Zelizer (1985) paints a similar picture arguing that during the late 19th and early 20th century a new social construct emerged in which the child was “*economically useless but emotionally priceless.*” This view is supported by the work of Duane Alwin who has conducted several studies of the changes in traits parents desire in their children. Employing data from Detroit Michigan from 1958 to 1983, Middletown Indiana from 1924 to 1978, as well as nationally representative data from 1964 to 1984, Alwin (1988, 1990 and 1989) concludes that “*child qualities generally thought to be associated with obedience or conformity (to obey parents, to have good manners, to be neat and clean, and to act according to sex-role norms) are seen to have declined in importance, and the qualities generally associated with autonomy or self-direction (good sense and sound judgment, honesty, responsible, and considerate) have increased in their assessed importance to parents during this period.*” The documented changes in attitudes suggest that parents have become increasingly attentive to the demands and desires of their children.

There is also evidence that suggests that parents have become less strict and discipline methods less severe. For example, we found two nationally representative surveys (from 1987 and 1995) in which parents were asked if they were more, less or equally as strict as their own parents. In both surveys, a greater percent of respondents said that they were less strict relative to their parents as opposed to more strict (see Figure 4). These views have also manifested in the attitudes towards corporal punishment. The General Social Survey (GSS) is a nationally representative survey that has collected parental attitudes towards discipline between 1986 and 2014 (Smith et al. 2014). Specifically, the GSS asks parents if they agree that “*it is sometimes necessary to discipline a child with a good, hard spanking?*” Figure 5

shows that the percent of parents that strongly agree or agree with this statement has declined from 84% in 1986 to 70% in 2014. While there is no long-term series of representative data on the incidence of corporal punishment within the household, there are some data on the use of physical discipline measures in public schools. Gershoff et al. (2015) document a steady decline in corporal punishment in schools in the latter half of the 20th century. Since 1970's 30 states have passed laws prohibiting the use of corporal punishment in schools (corporal punishment in public schools still remains legal in 19 states). Whether as a consequence of the laws or shifts in social norms, the prevalence of corporal punishment in schools dropped from 4% to 0.5% between 1978 and 2016 (Gershoff et al. 2015). Overall, the existing evidence indicates that, compared with previous generations, parents today are more lenient towards their children and view physical punishment as a less acceptable form of discipline for children.

In addition to changes in discipline, parents are spending more money on their children. Kornrich and Furstenberg (2013) use data from the Consumer Expenditure Survey (CES) and find that investment per child (in absolute terms and relative to overall consumption and income) increased between 1972 and 2007. Part of this change is driven by the fact that children are taking longer to become financially independent (see Danziger and Rouse 2007 and Furstenberg et al. 2004) and parents now allocate a significant portion of income to older children. For example, Schoeni and Ross (2005) document that parents allocate about 10% of their income to children older than 18. However, not all of this increase is driven by later stage investments and Kornrich and Furstenberg (2013) note that “*parents during the 1990s and 2000s spent earlier and extended their support for children into the later ages.*” Thus, by investing more money for longer periods of time, parents are increasingly allocating more financial resources to their children.

Lastly, parents also appear to be spending more time with their children. Bryant and Zick (1996) study time use data of mothers in Indiana between the 1920's and the 1980's. They find that, despite declines in family size, mothers have increased the total time with children and, hence, time per child has markedly increased. Bianchi et al. (2006) conduct a thorough analysis of nationally representative time use data to study changes in parental behavior between 1965 and 2001. They find that married parents are spending more time with their children even though they are working longer hours.⁴ Figure

⁴Aguiar and Hurst (2007) also find an increase in time allocated towards children but they interpret these results with caution as the measurement tool changed at the turn of the century. See Egerton, Fisher, and Gershuny (2006) for a

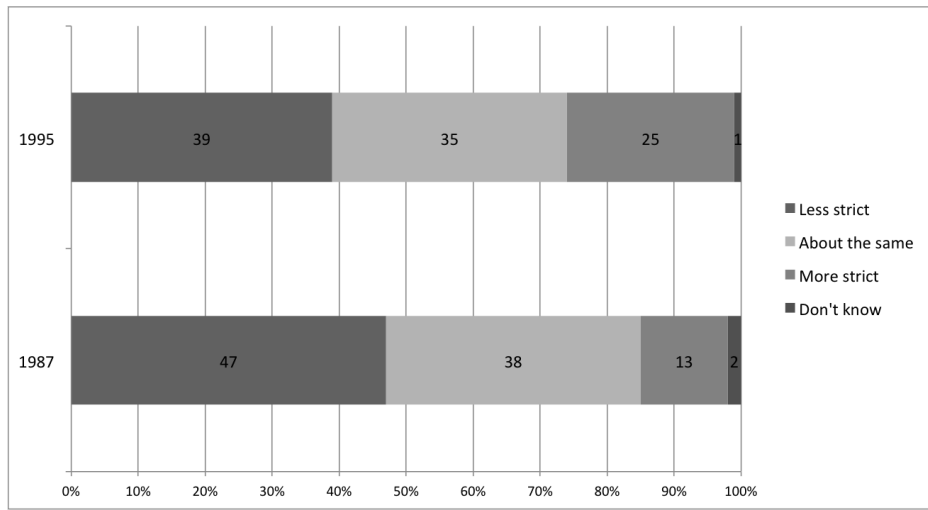
6 is taken from their study and shows that both married mothers and fathers have increased absolute time spent with children between 1975 and 2000. The evidence here is consistent with the narratives presented in Lareau (2003), who documents the measures parents take to organize their schedules around the activities of their children. In reference to this style of parenting, Bianchi et al. (2006) note that “*this middle-class vision of intensive parental involvement in children’s activities to ensure later success is spreading. It was not the norm, even for the middle class in 1965.*” The research discussed here indicates that parents are increasingly organizing their lives around the demands of their children.

Overall, the existing evidence on the evolution of parents’ attitudes and behaviors suggest that the relationship between parents and children shifted over the last century. We are not arguing that parents loved their children less in the past. Indeed, the historical evidence suggests that parents have loved and cared for their children throughout time.⁵ We are, however, arguing that the nature of the parent child relationship—as seen in the allocation of resources (both time and money) within the household, the use of corporal punishment and general attitudes—did seem to change in a way that benefited children.

discussion of the measurement issues.

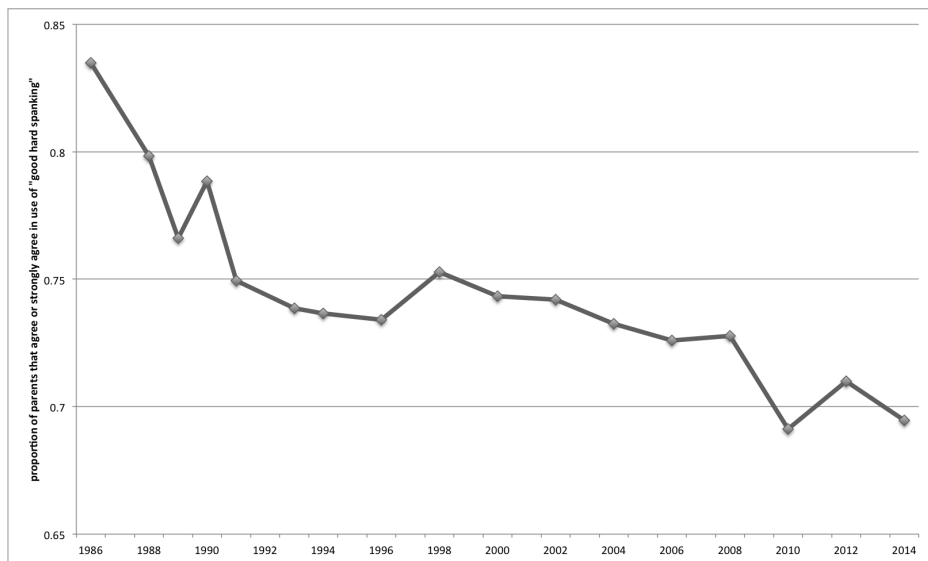
⁵There is an extensive historical literature examining parenting styles in Western society. In 1960 Philippe Ariés published *Centuries of Childhood* in which he argued that there was no concept of childhood in the middle ages. While historians have widely denounced many of his claims, this work is credited with setting the agenda for research on childhood over the past half century. A decade later, a number of researchers—DeMause (1974), Shorter (1976), Stone (1977), Plumb (1975) and Trumbach (1978)—provided evidence of a shift in attitudes towards children sometime between 1500 and modern day. The specific details and interpretation presented by these authors differ, but they all agree that this shift led to more affectionate relationships between children and their parents. This view came under fire in the 1980’s starting with Pollock (1983). She studies over 400 journals and autobiographies written between 1500 and 1900 and finds little evidence of systematic change in attitudes towards children. Given the serious data constraints (which Pollock discusses in detail), a consensus on parenting styles before the 20th century is unlikely to form. However, Hugh Cunningham (1995) reviews the existing literature and argues that, taken as a whole, the evidence does suggest that a shift in parenting styles took place over the last few centuries. He notes that while there are hints of this shift starting in the seventeenth century, the most drastic changes occurred in the 20th century.

Figure 4: Comparison of Parenting Style to Own Parents



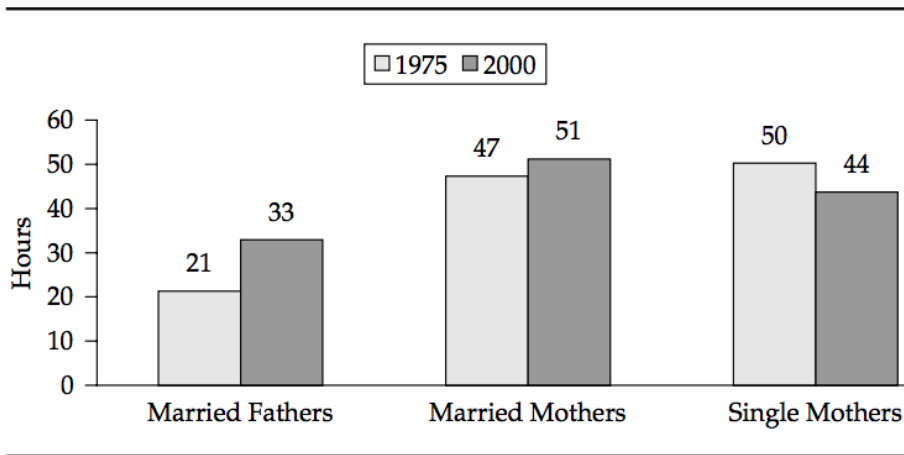
Note: Data from 1987 and 1995 are from the Parents Magazine Poll and Gallup Poll respectively.

Figure 5: Attitudes Towards Corporal Punishment in the Home



Note: Individual weights are applied to account for the fact that surveys are representative of households but only one adult per household responds.

Figure 6: Total Weekly Hours with Children



Source: Authors' calculations from the 1975–76 Time Use in Economic and Social Accounts and the combined file of the 1998–99 Family Interaction, Social Capital and Trends in Time Use Study and the 2000 National Survey of Parents.

Note: Difference between 2000 and 1975 estimates statistically significant at $p < 0.05$ for married fathers.

Note: Figure is taken from *Changing Rhythms of American Family Life* (Bianchi et al. 2006).

2.4 Summary of Empirical Trends

In this section we have documented three trends that occurred over the course of the 20th century: (1) wives entered the labor force and their contribution to household income rose dramatically, (2) the average number of children per household declined and (3) parenting styles shifted—as seen in changes in the allocation of resources (both time and money) within the household, the use of corporal punishment and general attitudes—in a way that allowed children to capture a greater share of the household surplus. We argue that there is a causal link between the first two trends and the changes in parenting styles. In the next section we develop a formal model of parenting styles in which competition between family members plays a central role in mediating this relationship.

3 A Simple Noncooperative Model of Intra-household Decisions

This section develops a model of intra-household decisions that capture parenting styles. Consider a household with two parents indexed by $p \in \{m, f\}$ and $N \geq 1$ children who are indexed by $c \in \{1, \dots, N\}$.

Let y^p denote the income of parent p , x^p his or her contribution to the material household consumption, i^p his or her material investment to influence the behavior of the children, and b^p the behavior of the children toward parent p . The budget constraint of parent p is $x^p + i^p = y^p$ and p 's utility is given by:

$$u^p = \left(\frac{x^p + \beta_x x^{-p}}{1 + \beta_x} \right)^{\alpha_x} (b^p)^{\alpha_b},$$

where $\alpha_x > 0$, $\alpha_b > 0$ and $\alpha_x + \alpha_b = 1$. The first term captures the value of material consumption and indicates that material consumption is partially a public good. The parameter $\beta_x \in [0, 1]$ governs the extent to which material consumption is a public good where $\beta_x = 1$ ($\beta_x = 0$) represents a purely public (private) good. The second term captures the behavior of the children toward parent p . As will be discussed below, b^p will depend on the number of children N as well as the investments made by each parent to influence the behavior of the children (i^p and i^{-p}).

Parents influence the behavior of the children by using a first-price auction to divide their material investments among the N children.⁶ Children submit bids that are interpreted as proposed levels of behavior. The idea is that children and parents have different preferences over the behavior of the children and by submitting higher bids, children agree to behave closer to the behavior preferred by the parents. As we will see, the children are willing to offer better behavior in return for higher levels of investments. The auction framework allows us to capture the idea that the household is an environment in which individuals compete with each other over limited resources. In particular, assume that the behavior of the children toward parent p is given by:

$$b^p = \sum_{c=1}^N \frac{\mathbf{E}(b_p^c) + \beta_b \mathbf{E}(b_{-p}^c)}{1 + \beta_b},$$

where $\mathbf{E}(b_p^c)$ is the expected behavior of child c in the auction run by parent p and $\sum_{c=1}^N \mathbf{E}(b_p^c)$ is the expected behavior of the children in the auction of parent p (this is equivalent to the expected revenue from the auction). Note that $\beta_b \in [0, 1]$, which means that the behavior of the children is in part a public good, but each parent cares relatively more about the behavior of the children in his or her own auction. We interpret this as parents having, to some extent, different preferences over the behavior of the children. For example, both parents benefit when the children put more effort at school regardless of

⁶We have also solved the model using an all-pay auction with no reserve price. The main results hold, but we require that $N \geq 2$.

which parent is inducing such behavior. At the same time, the love and respect of a child could be more of a private good for each parent.

The timing of events is as follows: (1) *Investment*. Parents choose investment levels and reservation prices in each auction. (2) *Auction*. For each auction, children's valuations are realized and they simultaneously and independently submit bids. The investment is allocated to the highest bidder and the highest bidder pays his or her bid to the parent.

Let $v_p^c \in [0, i^p]$ be the valuation that child c gives to i^p units of investment in p 's auction. Assume that v_p^c is independently and identically distributed and follows a uniform distribution on $[0, i^p]$. Child c observes his or her valuation but only knows that other siblings' values are independently and uniformly distributed on $[0, i^p]$. Let $b_p^c \in [0, i^p]$ denote the behavior that child c bids in p 's auction and suppose that parent p sets a reserve price equal to $r^p \in [0, i^p]$. Then, the utility that child c obtains from p 's auction is given by:

$$u_p^c = \begin{cases} v_p^c - b_p^c & \text{if } b_p^c > \max_{j \neq c} \{b_p^j, r^p\}, \\ 0 & \text{if } b_p^c < \max_{j \neq c} \{b_p^j, r^p\}. \end{cases}$$

We assume that the total utility of the child is simply the sum of the expected payoffs from each auction, i.e.,

$$u^c = \mathbf{E}(u_m^c) + \mathbf{E}(u_f^c).$$

4 Equilibrium Analysis

This section characterizes the equilibrium. We proceed through backward induction. First, we study the equilibrium in the auctions among children and use the notion of symmetric Bayesian Nash equilibrium. Second, given the equilibrium payoffs in the auction stage, we characterize the Nash equilibrium investments by the parents. Since the goal of the model is to explore how changes in the family size and the relative earnings of the spouses influence the distribution of resources within the household, at end of each stage of the backward induction procedure we discuss how household size and the relative income of the spouses affect the equilibrium.

4.1 Competition Among Siblings

When choosing their bids, children know their own valuation and the distribution of the valuations of their siblings. The following proposition characterizes the bid function equilibrium of the auction stage, the expected payoff of each child and the expected level of behavior derived from each auction.

Proposition 1 *Competition among siblings.* *The symmetric Bayesian Nash equilibrium bid function in the auction of parent p is given by:*

$$b(v_p^c) = \frac{(N-1)v_p^c + (r^p)^N (v_c^p)^{1-N}}{N} \text{ for } v_c^p \in [r^p, i^p].$$

The equilibrium expected behavior of the children in the auction of parent p is given by:

$$\sum_{c=1}^N \mathbf{E}(b_p^c) = \frac{(N-1)(i^p)^{N+1} + (N+1)(r^p)^N i^p - 2N(r^p)^{N+1}}{(i^p)^N (N+1)},$$

while the equilibrium expected payoff of a child in the action of parent p is given by:

$$\mathbf{E}(u_p^c) = \frac{(i^p)^{N+1} - (N+1)(r^p)^N i^p + N(r^p)^{N+1}}{(i^p)^N N(N+1)}.$$

Moreover, if parents can select any reserve price, in equilibrium, both choose $r^p = i^p/2$. Then:

$$\sum_{c=1}^N \mathbf{E}(b_p^c)(r^p = i^p/2) = f(N) i^p \text{ and } \mathbf{E}(u_p^c)(r^p = i^p/2) = g(N) i^p,$$

where $f(N) = \frac{(N-1)(2)^{N+1}}{(N+1)(2)^N}$ and $g(N) = \frac{(2)^{N+1} - (N+2)}{(2)^{N+1} N(N+1)}$.

Proof: see the Appendix. ■

Note that the bid function is increasing in the number of siblings. Formally, the derivative of $b(v_p^c)$ with respect to N is given by:

$$\frac{\partial b(v_p^c)}{\partial N} = \frac{(r^p)^N (v_c^p)^{1-N}}{N^2} \left[\left(\frac{v_c^p}{r^p} \right)^N - \ln \left(\frac{v_c^p}{r^p} \right)^N - 1 \right] > 0.$$

Thus, more children in the household lead to greater competition, which raises the bids offered by each child. Moreover, higher bids induce a lower expected payoff for each child, but better expected behavior

as experienced by the parent running the auction. Formally, the derivative of $\mathbf{E}(u_p^c)$ with respect to N is given by:

$$\frac{\partial \mathbf{E}(u_p^c)}{\partial N} = \left\{ \frac{-(2N+1) \left(\frac{i^p}{r^p}\right)^{N+1} + \left(\frac{i^p}{r^p}\right) (N+1)^2 \left[1 + \ln\left(\frac{i^p}{r^p}\right)^N\right] - N^2 \left[1 + \ln\left(\frac{i^p}{r^p}\right)^{N+1}\right]}{N^2 (N+1)^2 \left(\frac{i^p}{r^p}\right)^N} \right\} r^p < 0,$$

while the derivative of $\sum_{c=1}^N \mathbf{E}(b_p^c)$ with respect to N is given by:

$$\frac{\partial \sum_{c=1}^N \mathbf{E}(b_p^c)}{\partial N} = \left\{ \frac{2 \left(\frac{i^p}{r^p}\right)^{N+1} - (N+1)^2 \left(\frac{i^p}{r^p}\right) \ln\left(\frac{i^p}{r^p}\right) - 2(N+1) + 2N \left[1 + \ln\left(\frac{i^p}{r^p}\right)^{N+1}\right]}{(N+1)^2 \left(\frac{i^p}{r^p}\right)^N} \right\} r^p > 0.$$

Bids are also higher when the investment available in auction p is higher. Two mechanisms are behind this result. First, for a given reserve price, the distribution of valuations for a higher i^p first order stochastically dominates the distribution of valuations for a lower i^p (recall the valuations are uniformly distributed on the interval $[0, i^p]$). This induces children to increase their bids when i^p rises. Second, bids are increasing in the reserve price, which strengthens the effect of investment on bids since, in equilibrium, the reserve price is increasing in investment. Thus, children offer better behavior in return for higher rewards. Formally, $\sum_{c=1}^N \mathbf{E}(b_p^c) (r^p = i^p/2) = f(N) i^p$ and $\mathbf{E}(u_p^c) (r^p = i^p/2) = g(N) i^p$ are both increasing in i^p .

Summing up, the expected behavior of the children in the auction of parent p is increasing in the number of children and investment while the expected payoff of a child is decreasing in the number of children but increasing in investment.

4.2 Competition Between Parents

Parents know the distribution of the children's valuations and, hence, they can anticipate the equilibrium of the auction stage. Thus, employing Proposition 1, expected utility of parent p can be rewritten as follows:

$$u^p = \left[\frac{y^p - i^p + \beta_x (y^{-p} - i^{-p})}{1 + \beta_x} \right]^{\alpha_x} \left[\frac{f(N) (i^p + \beta_b i^{-p})}{1 + \beta_b} \right]^{\alpha_b}.$$

Each parent chooses a level of investment to maximize his or her own expected utility, taking the investment of the other spouse as given. A Nash equilibrium is a pair of investment decisions of the mother and father that are mutual best responses.

We make two simplifying assumptions. First, without loss of generality, we assume that $y^f \geq y^m > 0$. Second, we assume that $\beta_x < \alpha_x < \alpha_b/\beta_b$. As we shall see, the second assumption states that there are not large asymmetries regarding how parents value material consumption and the behavior of their children. The assumption guarantees that there is no equilibrium in which both parents play a corner solution. The following proposition fully characterizes the equilibrium investment choices.

Proposition 2 *Competition between parents.* *Suppose that $y^f \geq y^m > 0$ and $\beta_x < \alpha_x < \alpha_b/\beta_b$.*

1. *If $\beta_b > \beta_x$, then, the Nash equilibrium levels of investments are given by:*

$$i^m = \begin{cases} i^{m,*} & \text{if } y^m/y^f > \bar{y}^r, \\ 0 & \text{if } y^m/y^f \leq \bar{y}^r. \end{cases} \quad i^f = \begin{cases} i^{f,*} & \text{if } y^m/y^f > \bar{y}^r, \\ \alpha_b(y^f + \beta_x y^m) & \text{if } y^m/y^f \leq \bar{y}^r. \end{cases}$$

2. *If $\beta_b < \beta_x$, then, the Nash equilibrium levels of investments are given by:*

$$i^m = \begin{cases} i^{m,*} & \text{if } y^m/y^f > \hat{y}^r, \\ y^m & \text{if } y^m/y^f \leq \hat{y}^r. \end{cases} \quad i^f = \begin{cases} i^{f,*} & \text{if } y^m/y^f > \hat{y}^r, \\ \alpha_b y^f - \alpha_x \beta_b y^m & \text{if } y^m/y^f \leq \hat{y}^r, \end{cases}$$

where:

$$i^{m,*} = \alpha_b \left[(\delta_1 - \delta_2 \beta_x) y^m + (\delta_1 \beta_x - \delta_2) y^f \right], \quad i^{f,*} = \alpha_b \left[(\delta_1 - \delta_2 \beta_x) y^f + (\delta_1 \beta_x - \delta_2) y^m \right],$$

$$\bar{y}^r = \frac{\alpha_x (\beta_b - \beta_x)}{\alpha_x (1 - \beta_b \beta_x) + \alpha_b (1 - \beta_x \beta_x)}, \quad \hat{y}^r = \frac{(\beta_x - \beta_b) \alpha_b}{1 - (\alpha_b \beta_x + \alpha_x \beta_b) \beta_b},$$

$$\delta_1 = \frac{1}{1 - (\alpha_b \beta_x + \alpha_x \beta_b)^2}, \quad \delta_2 = \frac{\alpha_b \beta_x + \alpha_x \beta_b}{1 - (\alpha_b \beta_x + \alpha_x \beta_b)^2}.$$

Proof: see the Appendix. ■

Proposition 2 shows that the competition between the parents can adopt two forms. When $\beta_b > \beta_x$, for more traditional families ($y^m/y^f \leq \bar{y}^r$) only the father influences the behavior of the children, while for more modern families ($y^m/y^f > \bar{y}^r$) both parents influence the behavior of the children. The intuition

is as follows. When $\beta_b > \beta_x$, the public good component of the behavior of the children is dominant. Thus, if the mother is relatively poor with respect to the father, she prefers to free ride off the the father's investment. On the contrary, when $\beta_b < \beta_x$, the rivalry between the parents is dominant. The behavior of the children becomes predominantly a contest between the parents. In this case, in traditional families ($y^m/y^f \leq \hat{y}^r$), the mother invests all her income in the children, while in modern families ($y^m/y^f > \hat{y}^r$) both parents actively dispute to influence the behavior of the children.

The following corollary describes how parental income, β_x and β_b all interact to determine the share of resources that children are able to capture.

Corollary 1 *Investment shares.* *Under the assumptions in Proposition 2.*

1. If $\beta_b > \beta_x$, then:

$$\frac{i^m + i^f}{y^m + y^f} = \begin{cases} \alpha_b (1 + \beta_x) (\delta_1 - \delta_2) & \text{if } y^m/y^f > \bar{y}^r, \\ \frac{\alpha_b (y^f + \beta_x y^m)}{y^m + y^f} & \text{if } y^m/y^f \leq \bar{y}^r. \end{cases}$$

where $(i^m + i^f) / (y^m + y^f)$ is weakly decreasing in y^m/y^f .

2. If $\beta_b < \beta_x$, then:

$$\frac{i^m + i^f}{y^m + y^f} = \begin{cases} \alpha_b (1 + \beta_x) (\delta_1 - \delta_2) & \text{if } y^m/y^f > \hat{y}^r, \\ \frac{\alpha_b y^f + (1 - \alpha_x \beta_b) y^m}{y^m + y^f} & \text{if } y^m/y^f \leq \hat{y}^r. \end{cases}$$

where $(i^m + i^f) / (y^m + y^f)$ is weakly increasing in y^m/y^f .

Proof: see the Appendix. ■

When $\beta_b > \beta_x$, the behavior of the children is relatively more of a public good. Thus when $y^m/y^f < \bar{y}^r$ the husband increases his investment in the children when his wife's income increases. However, his increase in investment only accounts for his own utility while his wife continues to invest nothing in the children. Thus, the share of resources allocated towards the children decreases as the income gap between mother and father decreases. When $\beta_b < \beta_x$, more equal income between parents allows the children to capture a larger portion of the surplus. This prediction is better aligned with the historical evidence. In this case the behavior of the children is more of a private good. Thus, when $y^m/y^f < \hat{y}^r$ the wife will choose to free-ride off of the husband's contributions to household material goods while both the husband and wife will choose to compete for the behavior of the children. As the income of the wife increases, she

has more resources available to her to invest in the children. Thus, closing the gap between male and female earnings will result in more resources allocated to the children.

4.3 Distribution within the Household

The following corollary summarizes how the income of the household, the ratio of the income of the spouses and the number of children affect the distribution of payoffs within the household.

Corollary 2 *Distribution within the Household.* *Under the assumptions in Proposition 2. In addition, assume that $\beta_b < \beta_x$ and $y^m/y^f \leq \hat{y}^r$. Then:*

1. Nu^c/u^f is independent of $(y^m + y^f)$, increasing in y^m/y^f and decreasing in N for $N \geq 3$. Moreover, there exists $\bar{\alpha}_b$ such that if $\alpha_b \geq \bar{\alpha}_b$, then Nu^c/u^f is decreasing in N for all $N \geq 1$.
2. Nu^c/u^m is independent of $(y^m + y^f)$, decreasing in y^m/y^f for $y^m/y^f \leq \tilde{y}^r$ and increasing for $\tilde{y}^r \leq y^m/y^f \leq \hat{y}^r$, and decreasing in N for $N \geq 3$. Moreover, there exists $\bar{\alpha}_b$ such that if $\alpha_b \geq \bar{\alpha}_b$, then Nu^c/u^f is decreasing in N for all $N \geq 1$.
3. u^m/u^f is increasing in y^m/y^f and does not depend on $(y^m + y^f)$ or N .

Proof: see the Appendix. ■

Corollary 2 summarizes the three most important predictions of the model. First, the total household income does not affect the distribution of surplus. This highlights the fact that the baseline model abstracts from income effects and focuses on the role of competition. Mechanically, this result is driven by the assumption of homothetic preferences of parents and children. We can extend the model to incorporate income effects and in the following section we explore the interaction between total household income and discipline. Second, as the income-gap between the spouses closes, children improve their position relative to the father (Nu^c/u^f increases with y^m/y^f) and possibly also relative to the mother (Nu^c/u^m increases with y^m/y^f when $\tilde{y}^r \leq y^m/y^f \leq \hat{y}^r$). Third, as the number of siblings declines the combined utility of all children increases relative to each parents utility. In other words, in modern advanced societies with few kids and relatively more equal distribution of earnings between the parents, children are the rulers of the family.

5 Extension: Punishment

Beside investments (carrots), parents can also employ punishment (sticks) in order to influence the behavior of the children. Suppose that before parents decide their investment levels, the father has the ability to impose punishment to the children. Assume that the utility of parent p augmented with punishment is given by:

$$u^p = \left[\frac{y^p - i^p + \beta_x (y^m - i^m)}{1 + \beta_x} \right]^{\alpha_x} (b^p)^{\alpha_b} (e)^{-c_z^p z},$$

where z is the punishment that the father chooses to inflict upon the children and $c_z^p > 0$ is the marginal cost of inflicting punishment for parent p . Note that, ceteris paribus, parents would rather not punish their children since they themselves suffer from the use of punishment. However, punishment is a tool that could be used to influence the behavior of the children. In particular, assume that the behavior of the children toward parent p is given by:

$$b^p = \frac{\sum_{c=1}^N [\mathbf{E}(b_p^c) + \beta_b \mathbf{E}(b_{-p}^c)] + [\chi^p + (1 - \chi^p) \beta_b] (N)^\gamma (z)^{1-\gamma}}{1 + \beta_b},$$

where $\chi^p = 1$ if $p = f$ and $\chi^p = 0$ if $p = m$ and $\gamma \in (0, 1)$. Note that the marginal effect of z on b^p is increasing in the number of children, which captures the idea that it is more effective for the parents to influence the behavior of the children through punishment when they have more children (for example, when one child is punished, parents establish a reputation with other children). Finally, assume that the utility of child c augmented with punishment is given by

$$u^c = \mathbf{E}(u_m^c) + \mathbf{E}(u_f^c) - c_z^c z,$$

where $c_z^c > 0$ is the marginal cost of the punishment suffered by child c . The following proposition characterizes the equilibrium punishment and investment levels.

Proposition 3 *Suppose that $y^m/y^f \leq \hat{y}^r$, $\beta_b < \beta_x < \alpha_x < \alpha_b/\beta_b$ and $\bar{z} = \left[\frac{\alpha_b f(N)}{\alpha_x (N)^\gamma} \right]^{\frac{1}{1-\gamma}} \left(y^f - \frac{\alpha_x \beta_b}{\alpha_b} y^m \right)^{\frac{1}{1-\gamma}}$. Then, the Nash equilibrium levels of investments and punishment are*

given by:

$$i^m = y^m, i^f = \alpha_b y^f - \alpha_x \beta_b y^m - \alpha_x [f(N)]^{-1} (N)^\gamma (z)^{1-\gamma},$$

$$z = \begin{cases} \bar{z} & \text{if } \frac{(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(\bar{z}^f)^\gamma + (N)^\gamma (\bar{z}^f)} \geq c_z^f, \\ z^* & \text{if } \frac{(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(z^*)^\gamma + (N)^\gamma (z^*)} \leq c_z^f, \end{cases}$$

where z^* is the unique solution to:

$$\frac{(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(z^*)^\gamma + (N)^\gamma (z^*)} = c_z^f.$$

Proof: see the Appendix. ■

Several remarks apply. First, Proposition 3 expands the analysis of Proposition 2 when $\beta_b < \beta_x$ and $y^m/y^f \leq \hat{y}^r$, the region of the parameter space for which more equal income between spouses lead to a larger portion of the surplus allocated to the children. Second, investment (carrots) and punishment (sticks) are substitute inputs to produce good behavior. Indeed, as z increases, i^f decreases. Third, the father determines his optimal punishment level equalizing the marginal benefit of punishment with the marginal cost of punishment c_z^f . The marginal benefit of punishment for the father has two components. There is a direct effect of punishment on the behavior of the children and there is an indirect effect given by the impact of punishment on investment. As the father selects a higher level of punishment he invests less because carrots and sticks are substitute inputs. More importantly, the marginal benefit of punishment is affected by the income of each parent and the size of the household. The following corollary explores how equilibrium punishment changes with household income, the ratio of the spouses income and the number of children. It also shows that the introduction of punishment does not affect the key results deduced in Section 3.

Corollary 3 *Under the assumptions in Proposition 3.*

1. z^* is decreasing in $(y^f + y^m)$, increasing in y^m/y^f , and increasing (decreasing) in N if $\eta_{f,N} < \gamma$ ($\eta_{f,N} > \gamma$).
2. $(i^f + i^m) / (y^f + y^m)$ is increasing in y^m/y^f .

Proof: see the Appendix. ■

Corollary 3 provides three interesting new predictions. First, poorer families use punishment more intensively. The reason is that parents in poor families have fewer resources available to incentivize the children to behave well. Thus, they must resort to threat of punishment in order to influence the behavior of their children. This is similar to the intuition presented in Weinberg (2001). Second, as the gap between spousal earnings closes, the use of punishment is more likely. The reason is that the father has less income to influence the children and, hence, he is more willing to employ punishment. Third, larger families are more likely to use punishment when the elasticity of the expected behavior of the children in the auctions with respect to the number of children is relatively low. Finally, it is worth noting that introducing punishment into the model does not affect the key prediction that as the gap between male and female earnings closes more resources are allocated to the children.

6 Conclusion

In this paper we have developed a novel model of parenting and used it to explain the connection between improvements in economic opportunities for women, declines in family size and shifts in parenting styles over the course of the 20th century. We have argued that the rise in the relative earnings of wives increased competition between spouses for influencing the behavior of the children while the decline in family size reduced competition between children for resources from their parents. The combined effect improved the position of children within the household and allowed them to capture an increasing share of the household surplus over the past hundred years. In addition, we extended the model to incorporate punishment. We argued that an increase in total household income and a decrease in family size would reduce the use of punishment while a reduction in the earnings gap between husband and wife would increase the use of punishment. Empirically, we observe that the use of corporal punishment declined and we thus posit that the former two effects dominated the latter.

The main contribution of this paper is to highlight the importance of competition within the household. To date, there is very little academic work on this topic but there are good reasons to think that it plays an important role. For example, biological research has argued that within family competition explains a broad range of phenomenon including: begging behavior of birds (see Harper 1986), aggression of fig wasps (see West et al. 2001) and siderophore production in pathogenic bacteria (see Griffin et al.

2004). In addition, within family rivalries have appeared countless times in the arts (Shakespeare's *King Lear* offers a clear example). These forces are even present in religious texts. In the bible sibling rivalry plays out in a violent fashion between the first sons of mankind: Cain and Abel. Thus it seems reasonable to expect that competition within the household should shape behavior in meaningful ways.

Our effort to understand changes in parenting styles is largely motivated by the extensive empirical literature that documents the importance of parental inputs in determining long-term outcomes. Recent advances in our understanding of the skills formation process have pushed researchers to develop nuanced models of parent-child interactions. One important task of this literature is to explain the observed heterogeneity in parenting styles across time and space. We have argued that competition within the household is a useful concept for understanding this heterogeneity.

There are several promising routes to explore using the approach in this paper. First, it would be interesting to incorporate the different family structures that have become more common over the course of the 20th century. For example, one could introduce divorce in our model and study how competition between divorced parents affects the surplus obtained by the children. Second, since parenting styles are correlated with demographic and economic characteristics that are also associated with residential clustering, it may be productive to explore how parenting styles interact with the styles of other families in the community. For example, it is possible that more lenient parents produce a negative externality on their peers, who will find it more complicated to incentivize their children. Finally, since the economic literature on parenting styles is often motivated with concerns related to economic mobility, it would be interesting to extend this model to integrate external economic forces as well as the internal forces emphasized in this paper.

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Appendix to “When Children Rule: Parenting in Modern Families”

This appendix presents the proofs of all the propositions and corollaries.

A.1 Proof of Proposition 1

Suppose that $N \geq 1$. Let $b(v_p^c)$ be the bidding function and $\tilde{G}(v_p^c)$ the probability that child c wins the auction. Then, the expected utility of child c in the auction of parent p is $u_p^c = \int_0^{i^p} \tilde{G}(v_p^c) [v_p^c - b(v_p^c)] f(v_p^c) dv_p^c$, where $f(v_p^c) = (i^p)^{-1}$. The Bayesian Nash equilibrium bid function is $b(v_p^c) = [G(v_p^c)]^{-1} \left[r^p G(r^p) + \int_{r^p}^{v_p^c} yg(y) dy \right]$, where $G(v_p^c) = [F(v_p^c)]^{N-1}$, $g(v_p^c) = G'(v_p^c)$, $F(v_p^c) = v_p^c/i^p$, and r^p is the reserve price set by parent p (see Krishna 2010). Therefore:

$$\begin{aligned} b(v_p^c) &= [G(v_p^c)]^{-1} \left[r^p G(r^p) + \int_{r^p}^{v_p^c} yg(y) dy \right] \\ &= \left(\frac{i^p}{v_p^c} \right)^{N-1} \left[r^p \left(\frac{r^p}{i^p} \right)^{N-1} + (N-1) \int_{r^p}^{v_p^c} \left(\frac{y}{i^p} \right)^{N-1} dy \right] \\ &= \left(\frac{i^p}{v_p^c} \right)^{N-1} \left\{ r^p \left(\frac{r^p}{i^p} \right)^{N-1} + \frac{(N-1)}{N} \left(\frac{1}{i^p} \right)^{N-1} \left[(v_p^c)^N - (r^p)^N \right] \right\} \\ &= \frac{(N-1)v_p^c + (r^p)^N (v_p^c)^{1-N}}{N}. \end{aligned}$$

In equilibrium, the expected utility of a child in auction p is $\mathbf{E}(u_p^c) = \int_{r^p}^{i^p} G(v_p^c) [v_p^c - b(v_p^c)] f(v_p^c) dv_p^c$. Therefore:

$$\begin{aligned} \mathbf{E}(u_p^c) &= \int_{r^p}^{i^p} G(v_p^c) [v_p^c - b(v_p^c)] f(v_p^c) dv_p^c \\ &= \int_{r^p}^{i^p} \left(\frac{v_p^c}{i^p} \right)^{N-1} \left[v_p^c - \frac{(N-1)v_p^c + (r^p)^N (v_p^c)^{1-N}}{N} \right] \frac{1}{i^p} dv_p^c \\ &= \left(\frac{1}{i^p} \right)^N \int_{r^p}^{i^p} \frac{(v_p^c)^N - (r^p)^N}{N} dv_p^c \\ &= \frac{(i^p)^{N+1} - (N+1)(r^p)^N i^p + N(r^p)^{N+1}}{(i^p)^N N(N+1)}. \end{aligned}$$

In equilibrium, the expected behavior of children in the auction of parent p

is $\sum_{c=1}^N \mathbf{E}(b_p^c) = N \left\{ r^p [1 - F(r^p)] G(r^p) + \int_{r^p}^{i^p} v_p^c [1 - F(v_p^c)] g(v_p^c) dv_p^c \right\}$ or, equivalently, $\sum_{c=1}^N \mathbf{E}(b_p^c) = N \int_{r^p}^{i^p} b(v_p^c) [F(v_p^c)]^{N-1} f(v_p^c) dv_p^c$. Therefore:

$$\begin{aligned} \sum_{c=1}^N \mathbf{E}(b_p^c) &= N \int_{r^p}^{i^p} b(v_p^c) [F(v_p^c)]^{N-1} f(v_p^c) dv_p^c \\ &= N \int_{r^p}^{i^p} \left[\frac{(N-1)v_p^c + (r^p)^N (v_p^c)^{1-N}}{N} \right] \left(\frac{v_p^c}{i^p} \right)^{N-1} \frac{1}{i^p} dv_p^c \\ &= \frac{N}{(i^p)^N} \int_{r^p}^{i^p} \left[\frac{(N-1)(v_p^c)^N + (r^p)^N}{N} \right] dv_p^c \\ &= \frac{N}{(i^p)^N} \left[\frac{(N-1)(i^p)^{N+1}}{N(N+1)} + \frac{(r^p)^N i^p}{N} - \frac{2(r^p)^{N+1}}{(N+1)} \right]. \end{aligned}$$

Finally, in order to maximize $\sum_{c=1}^N \mathbf{E}(b_p^c)$, parent p must select $r^p = (1 - F(r^p)) / f(r^p)$, i.e., $r^p = i^p/2$. Introducing this into u_p^c and $\sum_{c=1}^N \mathbf{E}(b_p^c)$ we obtain $\sum_{c=1}^N \mathbf{E}(b_p^c) = f(N) i^p$ and $\mathbf{E}(u_p^c) = g(N) i^p$, where $f(N) = \frac{(N-1)(2)^{N+1} + 2}{(N+1)(2)^{N+1}}$ and $g(N) = \frac{2^{N+1} - (N+2)}{2^{N+1} N(N+1)}$. ■

A.2 Proof of Proposition 2

Employing Proposition 1, parent p expected utility can be rewritten as follows:

$$u^p = \left[\frac{y^p - i^p + \beta_x (y^{-p} - i^{-p})}{1 + \beta_x} \right]^{\alpha_x} \left[\frac{f(N) (i^p + \beta_b i^{-p})}{1 + \beta_b} \right]^{\alpha_b},$$

which is a twice continuously differentiable function for $(i^m, i^f) \in [0, y^m] \times [0, y^f]$. Then:

$$\begin{aligned} \frac{\partial u^p}{\partial i^p} &= u^p \left[\frac{-\alpha_x}{y^p - i^p + \beta_x (y^{-p} - i^{-p})} + \frac{\alpha_b}{i^p + \beta_b i^{-p}} \right], \\ \frac{\partial^2 u^p}{(\partial i^p)^2} &= -u^p \left\{ \frac{2 \frac{\alpha_x}{y^p - i^p + \beta_x (y^{-p} - i^{-p})} \frac{\alpha_b}{i^p + \beta_b i^{-p}}}{\left[\frac{\alpha_x \alpha_b}{[y^p - i^p + \beta_x (y^{-p} - i^{-p})]^2} + \frac{\alpha_b \alpha_x}{(i^p + \beta_b i^{-p})^2} \right]} \right\}. \end{aligned}$$

Note that $\partial^2 u^p / (\partial i^p)^2 < 0$, which implies that u^p is strictly concave in i^p . Therefore, from Glicksberg's theorem, there exists a pure strategy Nash equilibrium. Moreover, the reaction function of parent p is given by:

$$i^p (i^{-p}) = \begin{cases} 0 & \text{if } i^{-p} \geq \frac{\alpha_b(y^p + \beta_x y^{-p})}{(\alpha_b \beta_x + \alpha_x \beta_b)}, \\ \alpha_b (y^p + \beta_x y^{-p}) - (\alpha_b \beta_x + \alpha_x \beta_b) i^{-p} & \text{if } \frac{\alpha_b \beta_x y^{-p} - \alpha_x y^p}{(\alpha_b \beta_x + \alpha_x \beta_b)} < i^{-p} < \frac{\alpha_b (y^p + \beta_x y^{-p})}{(\alpha_b \beta_x + \alpha_x \beta_b)}, \\ y^p & \text{if } i^{-p} \leq \frac{\alpha_b \beta_x y^{-p} - \alpha_x y^p}{(\alpha_b \beta_x + \alpha_x \beta_b)}. \end{cases}$$

Before proceeding, we prove that under the assumption that $\beta_x < \alpha_x < \frac{\alpha_b}{\beta_b}$, the father always plays an interior solution. $i^f (i^m) \in (0, y^f)$ if and only if $\frac{\alpha_b \beta_x y^m - \alpha_x y^f}{(\alpha_b \beta_x + \alpha_x \beta_b)} < i^m < \frac{\alpha_b (y^f + \beta_x y^m)}{(\alpha_b \beta_x + \alpha_x \beta_b)}$. The first inequality always holds because $\beta_x < \alpha_x$ and $y^f \geq y^m$ implies $\alpha_b \beta_x y^m < \alpha_x y^f$. The second inequality always holds because $\alpha_x \beta_b < \alpha_b$ and $y^f \geq y^m$ implies $y^m \alpha_x \beta_b < \alpha_b y^f$, which implies $y^m < \frac{\alpha_b (y^f + \beta_x y^m)}{(\alpha_b \beta_x + \alpha_x \beta_b)}$. Since the father always plays an interior solution we must consider three possible situations.

(i) Suppose that $i^m = 0$. Then, $i^f = \alpha_b (y^f + \beta_x y^m)$. For this to be a Nash equilibrium, it must be the case that $i^f \geq \frac{\alpha_b (y^m + \beta_x y^f)}{(\alpha_b \beta_x + \alpha_x \beta_b)}$, i.e., whenever $\alpha_x (\beta_b - \beta_x) y^f \geq [\alpha_x (1 - \beta_b \beta_x) + \alpha_b (1 - \beta_x \beta_x)] y^m$. There is only one way in which this inequality can be satisfied, namely when $\beta_b > \beta_x$ and $\frac{y^m}{y^f} \leq \bar{y}^r = \frac{\alpha_x (\beta_b - \beta_x)}{\alpha_x (1 - \beta_b \beta_x) + \alpha_b (1 - \beta_x \beta_x)}$.

(ii) Suppose that $i^m = y^m$. Then, $i^f = \alpha_b y^f - \alpha_x \beta_b y^m$. For this to be a Nash equilibrium, $i^f \leq \frac{\alpha_b \beta_x y^f - \alpha_x y^m}{(\alpha_b \beta_x + \alpha_x \beta_b)}$, i.e., whenever $[1 - (\alpha_b \beta_x + \alpha_x \beta_b) \beta_b] \alpha_x y^m \leq \alpha_x (\beta_x - \beta_b) \alpha_b y^f$. There is only one way in which this inequality, namely, $\beta_x > \beta_b$ and $\frac{y^m}{y^f} \leq \hat{y}^r = \frac{(\beta_x - \beta_b) \alpha_b}{1 - (\alpha_b \beta_x + \alpha_x \beta_b) \beta_b}$.

(iii) Suppose that $i^m \in (0, y^m)$. Then:

$$\begin{aligned} i^m &= i^{m,*} = \alpha_b \left[(\delta_1 - \beta_x \delta_2) y^m + (\beta_x \delta_1 - \delta_2) y^f \right], \\ i^f &= i^{f,*} = \alpha_b \left[(\delta_1 - \beta_x \delta_2) y^f + (\beta_x \delta_1 - \delta_2) y^m \right], \end{aligned}$$

where $\delta_1 = \frac{(\alpha_x + \alpha_b)}{1 - (\alpha_b \beta_x + \alpha_x \beta_b)^2}$ and $\delta_2 = \frac{\alpha_b \beta_x + \alpha_x \beta_b}{1 - (\alpha_b \beta_x + \alpha_x \beta_b)^2}$. For this to be a Nash equilibrium $\frac{\alpha_b \beta_x y^f - \alpha_x y^m}{(\alpha_b \beta_x + \alpha_x \beta_b)} < i^f < \frac{\alpha_b (y^m + \beta_x y^f)}{(\alpha_b \beta_x + \alpha_x \beta_b)}$, i.e., whenever $\frac{\alpha_b \beta_x y^f - \alpha_x y^m}{(\alpha_b \beta_x + \alpha_x \beta_b)} < \alpha_b [(\delta_1 - \delta_2 \beta_x) y^f + (\delta_1 \beta_x - \delta_2) y^m] < \frac{\alpha_b (y^m + \beta_x y^f)}{(\alpha_b \beta_x + \alpha_x \beta_b)}$.

These inequality can be rewritten as follows

$$\begin{aligned} [(\alpha_b \beta_x + \alpha_x \beta_b) (\delta_1 \beta_x - \delta_2) - 1] y^m &< [\beta_x - (\alpha_b \beta_x + \alpha_x \beta_b) (\delta_1 - \delta_2 \beta_x)] y^f \\ \alpha_b [\beta_x - (\alpha_b \beta_x + \alpha_x \beta_b) (\delta_1 - \delta_2 \beta_x)] y^f &< [\alpha_x + \alpha_b (\alpha_b \beta_x + \alpha_x \beta_b) (\delta_1 \beta_x - \delta_2)] y^m \end{aligned}$$

Suppose that $\beta_b > \beta_x$. Then, $(\alpha_b \beta_x + \alpha_x \beta_b) (\delta_1 \beta_x - \delta_2) < 1$ and $\beta_x < (\alpha_b \beta_x + \alpha_x \beta_b) (\delta_1 - \delta_2 \beta_x)$. Then, the second inequality always holds, while the first holds if and only if:

$$\frac{y^m}{y^f} > \frac{(\alpha_b \beta_x + \alpha_x \beta_b) (\delta_1 - \delta_2 \beta_x) - \beta_x}{1 - (\alpha_b \beta_x + \alpha_x \beta_b) (\delta_1 \beta_x - \delta_2)} = \bar{y}^r$$

Suppose that $\beta_b < \beta_x$. Then, $(\alpha_b\beta_x + \alpha_x\beta_b)(\delta_1\beta_x - \delta_2) - 1 < \beta_x - (\alpha_b\beta_x + \alpha_x\beta_b)(\delta_1 - \delta_2\beta_x)$ and $\beta_x > (\alpha_b\beta_x + \alpha_x\beta_b)(\delta_1 - \delta_2\beta_x)$. Then, the first inequality always holds (recall that $y^f \geq y^m$), while the second inequality holds if and only if:

$$\frac{y^m}{y^f} > \frac{\alpha_b [\beta_x - (\alpha_b\beta_x + \alpha_x\beta_b)(\delta_1 - \delta_2\beta_x)]}{\alpha_x + \alpha_b (\alpha_b\beta_x + \alpha_x\beta_b)(\delta_1\beta_x - \delta_2)} = \hat{y}^r.$$

Summing up, we have the following two possible cases.

Case 1: $\beta_b > \beta_x$. Then, $(i^m, i^f) = (0, \alpha_b (y^f + \beta_x y^m))$ is the unique Nash equilibrium for $y^m/y^f \leq \bar{y}^r$, while $(i^m, i^f) = (i^{m,*}, i^{f,*})$ is the unique Nash equilibrium when $y^m/y^f > \bar{y}^r$.

Case 2: $\beta_b < \beta_x$. $(i^m, i^f) = (y^m, \alpha_b y^f - \alpha_x \beta_b y^m)$ is the unique Nash equilibrium when $y^m/y^f \leq \hat{y}^r$ and $(i^m, i^f) = (i^{m,*}, i^{f,*})$ is the unique Nash equilibrium when $y^m/y^f > \hat{y}^r$. ■

A.3 Proof of Corollary 1

Case 1: $\beta_b > \beta_x$. From Proposition 1, in equilibrium, $i^m + i^f = \alpha_b (y^f + \beta_x y^m)$ when $y^m/y^f \leq \bar{y}^r$, while $i^m + i^f = i^{m,*} + i^{f,*} = \alpha_b (1 + \beta_x) (\delta_1 - \delta_2) (y^m + y^f)$ when $y^m/y^f > \bar{y}^r$. Thus:

$$\frac{i^m + i^f}{y^m + y^f} = \begin{cases} \alpha_b (1 + \beta_x) (\delta_1 - \delta_2) & \text{if } y^m/y^f > \bar{y}^r, \\ \frac{\alpha_b (y^f + \beta_x y^m)}{(y^m + y^f)} & \text{if } y^m/y^f \leq \bar{y}^r. \end{cases}$$

Moreover, for $y^m/y^f \leq \bar{y}^r$, the derivative of $\frac{i^m + i^f}{y^m + y^f}$ with respect to y^m/y^f is given by:

$$\frac{d[(i^m + i^f) / (y^m + y^f)]}{d(y^m/y^f)} = \frac{-(1 - \beta_x)}{(1 + y^m/y^f)^2} < 0.$$

Also note that $\alpha_b (1 + \beta_x) (\delta_1 - \delta_2) = \frac{\alpha_b (1 + \beta_x \bar{y}^r)}{(1 + \bar{y}^r)}$. Thus, $(i^m + i^f) / (y^m + y^f)$ is weakly decreasing in y^m/y^f .

Case 2: $\beta_b < \beta_x$. From Proposition 1, in equilibrium, $i^m + i^f = \alpha_b y^f + [\alpha_x (1 - \beta_b) + \alpha_b] y^m$ when $y^m/y^f \leq \hat{y}^r$, while $i^m + i^f = i^{m,*} + i^{f,*}$ when $y^m/y^f > \hat{y}^r$. Thus:

$$\frac{i^m + i^f}{y^m + y^f} = \begin{cases} \alpha_b (1 + \beta_x) (\delta_1 - \delta_2) & \text{if } y^m/y^f > \hat{y}^r, \\ \frac{\alpha_b y^f + [\alpha_x (1 - \beta_b) + \alpha_b] y^m}{(y^m + y^f)} & \text{if } y^m/y^f \leq \hat{y}^r. \end{cases}$$

Moreover, for $y^m/y^f \leq \hat{y}^r$, the derivative of $\frac{i^m + i^f}{y^m + y^f}$ with respect to y^m/y^f is given by:

$$\frac{d[(i^m + i^f) / (y^m + y^f)]}{d(y^m/y^f)} = \frac{\alpha_x (1 - \beta_b)}{(y^m/y^f + 1)^2} > 0.$$

Also note that $\alpha_b(1 + \beta_x)(\delta_1 - \delta_2) = \frac{\alpha_b + [\alpha_x(1 - \beta_b) + \alpha_b]\hat{y}^r}{(1 + \hat{y}^r)}$. Thus, $(i^m + i^f) / (y^m + y^f)$ is weakly increasing in y^m/y^f . ■

A.4 Proof of Corollary 2

Under the assumptions in Proposition 2, when $\beta_b < \beta_x$ and $y^m/y^f \leq \hat{y}^r$, in equilibrium, Nu^c , u^f and u^m are given by:

$$\begin{aligned} Nu^c &= Ng(N) \left[\frac{\alpha_b + (1 - \alpha_x \beta_b) y^m/y^f}{1 + y^m/y^f} \right] (y^f + y^m), \\ u^f &= \left(\frac{\alpha_x}{1 + \beta_x} \right)^{\alpha_x} \left(\frac{f(N) \alpha_b}{1 + \beta_b} \right)^{\alpha_b} \left(\frac{1 + \beta_b y^m/y^f}{1 + y^m/y^f} \right) (y^f + y^m), \\ u^m &= \left(\frac{\alpha_x \beta_x}{1 + \beta_x} \right)^{\alpha_x} \left(\frac{f(N)}{1 + \beta_b} \right)^{\alpha_b} \left(\frac{1 + \beta_b y^m/y^f}{1 + y^m/y^f} \right)^{\alpha_x} \left[\frac{\beta_b \alpha_b + (1 - \alpha_x (\beta_b)^2) y^m/y^f}{1 + y^m/y^f} \right]^{\alpha_b} (y^f + y^m), \end{aligned}$$

respectively. Therefore, in equilibrium, the ratio between the payoff of the children and the father is given by:

$$\frac{Nu^c}{u^f} = \frac{Ng(N) [\alpha_b + (1 - \alpha_x \beta_b) y^m/y^f]}{\left(\frac{\alpha_x}{1 + \beta_x} \right)^{\alpha_x} \left(\frac{f(N) \alpha_b}{1 + \beta_b} \right)^{\alpha_b} (1 + \beta_b y^m/y^f)}.$$

From a direct inspection of this expression, it is easy to verify that Nu^c/u^f is independent of $(y^m + y^f)$. The derivative of Nu^c/u^f with respect to y^m/y^f is positive. Thus, Nu^c/u^f is increasing in y^m/y^f . The derivative of Nu^c/u^f with respect to N is negative if and only if

$$\left[\frac{(2)^{N+1} \ln 2 - 1}{(2)^{N+1} - (N + 2)} \right] < \alpha_b \left[\frac{(2)^{N+1} + (N - 1)(2)^{N+1} \ln(2)}{(N - 1)(2)^{N+1} + 2} \right] + (1 - \alpha_b) \left[\frac{1 + (N + 1) \ln 2}{N + 1} \right]$$

For $N \geq 1$, the left hand side of this inequality is always strictly lower than the first squared bracket on the right hand side. The left hand side is also strictly lower than the second squared bracket on the right hand side for $1 \leq N < \hat{N} \in (2, 3)$, where $(2)^{\hat{N}+1} = 1 + (\hat{N} + 2) (\hat{N} + 1) \ln(2)$, while the opposite holds when $N > \hat{N}$. Therefore, for $N \geq \hat{N}$, $\partial(Nu^c/u^f)/\partial N < 0$ and, hence, Nu^c/u^f is decreasing in N . For $N < \hat{N}$, the left hand side is strictly greater than the second squared bracket on the right hand side. Since the right hand side is just a weighted average of $\left[\frac{(2)^{N+1} + (N-1)(2)^{N+1} \ln(2)}{(N-1)(2)^{N+1} + 2} \right]$ and $\left[\frac{1 + (N+1) \ln 2}{N+1} \right]$, a simple application of the intermediate value theorem implies that for each $N < \hat{N}$ there must exist $\bar{\alpha}_b(N) \in (0, 1)$ such that for all $\alpha_b \geq \bar{\alpha}_b(N)$ the inequality holds. Define $\bar{\alpha}_b = \max_{N \in [1, \hat{N}]} \bar{\alpha}_b(N)$. Then, when $\alpha_b \geq \bar{\alpha}_b$, $\partial(Nu^c/u^f)/\partial N < 0$ for all $N \geq 1$ and, hence, Nu^c/u^f is decreasing for all $N \geq 1$.

In equilibrium, the ratio between the payoff of the children and the mother is given by:

$$\frac{Nu^c}{u^m} = \frac{Ng(N) [\alpha_b + (1 - \alpha_x \beta_b) (y^m/y^f)]}{\left(\frac{\alpha_x \beta_x}{1 + \beta_x}\right)^{\alpha_x} \left(\frac{f(N)}{1 + \beta_b}\right)^{\alpha_b} [1 + \beta_b (y^m/y^f)]^{\alpha_x} \left\{ \beta_b \alpha_b + [1 - (\beta_b)^2 \alpha_x] (y^m/y^f) \right\}^{\alpha_b}}$$

From a direct inspection of this expression, it is easy to verify that Nu^c/u^m is independent of $(y^m + y^f)$. The derivative of Nu^c/u^m with respect to y^m/y^f is positive (negative) if and only if $y^m/y^f > \tilde{y}^r$ ($y^m/y^f < \tilde{y}^r$), where:

$$\tilde{y}^r = \frac{(1 - \beta_b)^2 (1 - \alpha_x) + (\alpha_x)^2 (1 + \beta_b)}{\alpha_x - \beta_b + \alpha_x \beta_b - (\alpha_x)^2 \beta_b (1 - \beta_b) (1 + \beta_b)} - 1.$$

Thus, Nu^c/u^f decreasing in y^m/y^f for $y^m/y^f \leq \tilde{y}^r$ and increasing for $\tilde{y}^r \leq y^m/y^f \leq \hat{y}^r$. Finally, note that the sign of the derivative of Nu^c/u^m with respect to N is the same as the sign of the derivative of Nu^c/u^f with respect to N . Hence, the same proof that employed to sign $\partial(Nu^c/u^f)/\partial N$ applies to $\partial(Nu^c/u^m)/\partial N$.

In equilibrium, the ratio between the payoff of the mother and the father is given by:

$$\frac{u^m}{u^f} = \left(\frac{\beta_x}{\alpha_x}\right)^{\alpha_x} \left\{ \frac{\beta_b \alpha_b + [1 - \alpha_x (\beta_b)^2] y^m/y^f}{\alpha_b + \alpha_b \beta_b y^m/y^f} \right\}^{\alpha_b},$$

From a direct inspection of this expression, it is easy to verify that u^m/u^f does not depend on $(y^m + y^f)$ or N . The derivative of u^m/u^f with respect to y^m/y^f is positive. Thus, u^m/u^f is increasing in y^m/y^f .

■

A.5 Proof of Proposition 3

Suppose that the father has selected the punishment level $z^f \in [0, \bar{z}^f]$. Then, employing the same steps we use in the proof of Proposition 2, the reaction function of parent p is given by:

$$i^p(i^{-p}) = \begin{cases} 0 & \text{if } i^{-p} \geq \frac{\alpha_b(y^p + \beta_x y^{-p})}{(\alpha_b \beta_x + \alpha_x \beta_b)} - \frac{\alpha_x(N)^\gamma (z^p)^{1-\gamma}}{(\alpha_b \beta_x + \alpha_x \beta_b) f(N)}, \\ \alpha_b(y^p + \beta_x y^{-p}) - (\alpha_b \beta_x + \alpha_x \beta_b) i^{-p} & \text{if } \frac{\alpha_b \beta_x y^{-p} - \alpha_x y^p}{(\alpha_b \beta_x + \alpha_x \beta_b)} - \frac{\alpha_x(N)^\gamma (z^p)^{1-\gamma}}{f(N)(\alpha_b \beta_x + \alpha_x \beta_b)} < i^{-p} \\ -\frac{\alpha_x(z^p + \beta_b z^{-p})}{f(N)} & \text{\& } i^{-p} < \frac{\alpha_b(y^p + \beta_x y^{-p})}{(\alpha_b \beta_x + \alpha_x \beta_b)} - \frac{\alpha_x(N)^\gamma (z^p)^{1-\gamma}}{(\alpha_b \beta_x + \alpha_x \beta_b) f(N)}, \\ y^p & \text{if } i^{-p} \leq \frac{\alpha_b \beta_x y^{-p} - \alpha_x y^p}{(\alpha_b \beta_x + \alpha_x \beta_b)} - \frac{\alpha_x(N)^\gamma (z^p)^{1-\gamma}}{f(N)(\alpha_b \beta_x + \alpha_x \beta_b)}. \end{cases}$$

Note that $i^f(i^m) \in (0, y^f)$ if and only if $\frac{\alpha_b \beta_x y^m - \alpha_x y^f}{(\alpha_b \beta_x + \alpha_x \beta_b)} - \frac{\alpha_x(N)^\gamma (z^p)^{1-\gamma}}{f(N)(\alpha_b \beta_x + \alpha_x \beta_b)} < i^m < \frac{\alpha_b(y^f + \beta_x y^m)}{(\alpha_b \beta_x + \alpha_x \beta_b)} - \frac{\alpha_x(N)^\gamma (z^p)^{1-\gamma}}{(\alpha_b \beta_x + \alpha_x \beta_b) f(N)}$. The first inequality always holds because $\beta_x < \alpha_x$ and $y^f \geq y^m$ implies $\alpha_b \beta_x y^m < \alpha_x y^f$.

The second inequality holds whenever $z^f < \bar{z}^f = \left[\frac{\alpha_b f(N)}{\alpha_x (N)^\gamma} \right]^{\frac{1}{1-\gamma}} \left(y^f - \frac{\alpha_x \beta_b}{\alpha_b} y^m \right)^{\frac{1}{1-\gamma}}$. Moreover, note that $\alpha_x \beta_b < \alpha_b$ and $y^f \geq y^m$ implies $\bar{z}^f > 0$. Thus, if $\beta_x < \alpha_x < \alpha_b / \beta_b$, $y^f \geq y^m$, and $z^f < \bar{z}^f$, the father always plays an interior solution.

Suppose that $i^m = y^m$. Then, $i^f = i^f(y^m) = \alpha_b y^f - \alpha_x \beta_b y^m - \alpha_x [f(N)]^{-1} (N)^\gamma (z^f)^{1-\gamma}$. For this to be a Nash equilibrium of the investment subgame it must be the case that $i^f \leq \frac{\alpha_b \beta_x y^f - \alpha_x y^m}{(\alpha_b \beta_x + \alpha_x \beta_b)} - \frac{\alpha_x \beta_b z^f}{f(N)(\alpha_b \beta_x + \alpha_x \beta_b)}$, i.e., $\frac{y^m}{y^f} \leq \frac{1}{\hat{y}^r} + \frac{\alpha_b(\beta_x - \beta_b)(N)^\gamma (z^f)^{1-\gamma}}{[1 - (\alpha_b \beta_x + \alpha_x \beta_b)\beta_b]f(N)y^f}$. Since $\beta_x > \beta_b$ and $\frac{y^m}{y^f} \leq \hat{y}^r$, the last inequality holds for all z^f .

Thus, when $\beta_x > \beta_b$ and $\frac{y^m}{y^f} \leq \hat{y}^r$, the payoff of the father as a function of z^f is given by:

$$\ln u^f = \alpha_x \ln \left(\frac{y^f - i^f}{1 + \beta_x} \right) + \alpha_b \ln \left[\frac{f(N) i^f + \beta_b f(N) y^m + (N)^\gamma (z^f)^{1-\gamma}}{1 + \beta_b} \right] - c_z^f z^f,$$

where $i^f = \alpha_b y^f - \alpha_x \beta_b y^m - \alpha_x [f(N)]^{-1} (N)^\gamma (z^f)^{1-\gamma}$. Then, $\frac{\partial \ln u^f}{\partial z^f} = \frac{(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(z^f)^\gamma + (N)^\gamma z^f} - c_z^f$ and $\frac{\partial^2 \ln u^f}{(\partial z^f)^2} < 0$. Since $\lim_{z^f \rightarrow 0} \frac{\partial \ln u^f}{\partial z^f} > 0$, the best alternative for the father is to choose

$$z^f = \begin{cases} \bar{z}^f & \text{if } \frac{(\alpha_x + \alpha_b)(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(\bar{z}^f)^\gamma + (N)^\gamma \bar{z}^f} \geq c_z^f, \\ z^{f,*} & \text{if } \frac{(\alpha_x + \alpha_b)(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(\bar{z}^f)^\gamma + (N)^\gamma \bar{z}^f} \leq c_z^f, \end{cases}$$

where $z^{f,*}$ is the unique solution to $\frac{(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(z^{f,*})^\gamma + (N)^\gamma z^{f,*}} = c_z^f$. ■

A.6 Proof of Corollary 3

Under the assumption in Proposition 3, when $\frac{(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(\bar{z}^f)^\gamma + (N)^\gamma \bar{z}^f} \leq c_z^f$, the equilibrium level of punishment is given by the unique solution to $\frac{(1-\gamma)(N)^\gamma}{f(N)(y^f + \beta_b y^m)(z^{f,*})^\gamma + (N)^\gamma z^{f,*}} = c_z^f$. Let $\eta_{a,b}$ denote the elasticity of a with respect to a . A direct application of the implicit function theorem implies:

$$\begin{aligned} \eta_{z^{f,*},y} &= \frac{-f(N)(y^f + \beta_b y^m)}{\gamma f(N)(y^f + \beta_b y^m) + (N)^\gamma (z^{f,*})^{1-\gamma}} < 0, \\ \eta_{z^{f,*},y^m/y^f} &= \frac{(1 - \beta_b) f(N)(y^f + y^m)}{f(N)(y^f + \beta_b y^m)\gamma + (N)^\gamma (z^{f,*})^{1-\gamma}} \frac{(y^m/y^f)}{(1 + y^m/y^f)^2} > 0, \\ \eta_{z^{f,*},N} &= \frac{(\gamma - \eta_{f,N}) f(N)(y^f + \beta_b y^m)}{\gamma f(N)(y^f + \beta_b y^m)(z^{f,*})^{2(\gamma-1)} + (N)^\gamma (z^{f,*})^{\gamma-1}}, \end{aligned}$$

where $\eta_{z^{f,*},N} > 0$ if and only if $\gamma > \eta_{f,N}$.

Introducing $z^{f,*}$ into i^f , we obtain:

$$\frac{i^f + i^m}{y^f + y^m} = \frac{\alpha_b + (1 - \alpha_x \beta_b) y^m / y^f}{1 + y^m / y^f} - \frac{\alpha_x [f(N)]^{-1} (N)^\gamma (z^{f,*})^{1-\gamma}}{y^f + y^m}$$

Therefore:

$$\frac{\partial \left(\frac{i^f + i^m}{y^f + y^m} \right)}{\partial (y^m / y^f)} = \left[\frac{f(N) (y^f + \beta_b y^m) \gamma + (N)^\gamma \gamma (z^{f,*})^{1-\gamma}}{f(N) (y^f + \beta_b y^m) \gamma + (N)^\gamma (z^{f,*})^{1-\gamma}} \right] \frac{\alpha_x (1 - \beta_b)}{(1 + y^m / y^f)^2} > 0.$$

Thus, $(i^f + i^m) / (y^f + y^m)$ is increasing in y^m / y^f . ■